
Submissions: This assignment is due on T September 17th. Please note:

1. Each student must submit his or her own assignment.
2. Solutions should preferably be typed in Latex, MSWord or other such word processing software, or printed clearly. In either case, submit a hard copy of your solution.
3. You must write your name and UUID clearly on your submitted assignment.
4. Staple the pages together. Write your name and UUID on top of each page.
5. It is preferable that you submit solutions to me in class, but it is ok if you can’t do that - just make sure to submit solutions by end of day (i.e., 11:59 PM) by sliding it under my office door (DH 307).

Academic Integrity: You are encouraged to work in groups, but everyone must write out their own solutions. Absolutely no word to word copying is allowed. If you have worked with other students on the assignment or referred to external sources, please mention all names and sources on your assignment.

Partial solutions: Document your efforts at solving a problem even if you cannot solve it. Write why your approach failed.

Problem 1 [20 pts]: This is a problem from Jeff’s notes - reproduced here for ease. The $d$-dimensional hypercube is the graph defined as follows. There are $2^d$ vertices, each labeled with a different string of $d$ bits. Two vertices are joined by an edge if and only if their labels differ in exactly one bit. See figures in Jeff’s notes if you need to - but it would be more instructive to draw them yourself and recognize these objects. Recall that a Hamiltonian cycle is a closed walk that visits each vertex in a graph exactly once. Prove that for every integer $d \geq 2$, the $d$-dimensional hypercube has a Hamiltonian cycle.

Problem 2 [30 pts]: Problem 2.22 from the textbook.

Problem 3 [30 pts]: Problem 2.32 from the textbook.

Problem 4 [4 + 8 + 8 = 20 pts]: This is a problem from Jeff’s notes - reproduced here for ease. Suppose we are given a bit string $B[1..n]$. A triple of distinct indices $1 \leq i < j < k \leq n$ is called a well-spaced triple in $B$ if $B[i] = B[j] = B[k] = 1$ and $k - j = j - i$.

(a) Describe a brute-force algorithm to determine whether $B$ has a well-spaced triple in $O(n^2)$ time.
(b) Describe an algorithm to determine whether $B$ has a well spaced triple in $O(n \log n)$ time.
(c) Describe an algorithm to determine the number of well-spaced triples in $B$ in $O(n \log n)$ time.