Comp 2700 (Discrete Structures) Fall 2019. Homework 1.

Submissions: This assignment is due in class on T Sep 17th 2019. Each student must submit his or her own assignment. Solutions can either be typed in Latex, MSWord or other such word processing software, or printed clearly. You must write your name and UUID clearly on your submitted assignment.

Academic Integrity: You are encouraged to work in groups, but everyone must write out their own solutions. Absolutely no word to word copying is allowed. Please refer to the course policies and schedules about this. If you have worked with other students on the assignment or referred to external sources, please mention all names and sources on your assignment.

Partial solutions: If you are sure that you know how to arrive at a solution, but you get stuck in some place, it is better to write the partial solution. Honest attempts at partial solutions will be awarded.

Name:	
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Problem 1 [5 x 4 = 20 pts]: For each of the following equations, either give an argument why it is always true or show example sets which may falsify it:

(I) $A - (B \cap C) = (A - B) \cap (A - C)$.

(II) $(A \cap B) \cap (A \cap C) = A \cap (B \cap C)$.

(III) $2^{A \cap B} = 2^A \cap 2^B$.

(IV) $(A - B) \cup (A - C) = A - (B \cup C)$.

Problem 2 [10 pts]: Let A be the set of all integers which are 3 more than some perfect square. Examples of such integers are : $4 = 1 + 3$, $7 = 4 + 3$, $3 = 0 + 3$, $28 = 25 + 3$, but of course there are infinitely many of them. Write out the set A as succintly as you can, using the set-builder notation.

Problem 3 [20 pts]: Show that $1 + 3 + 5 + \ldots + (2n - 1) = n^2$ for every positive integer n.

Problem 4 [10 x 3 = 30 pts]: Express the following succintly using Σ and Π notations:

(I) $1 + 2 + 4 + 8 + \ldots + 2^n = 2^{n+1} - 1$.

(II) $1 \times 2 \times 3 \times 4 \times \ldots \times n = n!$. The $n!$ is to be read as "n factorial", and it is the product of all numbers from 1 through n.

(III) $\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \ldots + \frac{1}{n \cdot (n+1)} = 1 - \frac{1}{n+1}$. The $\cdot$ stands for multiplication.

Problem 5 [20 pts]: n people attend a party and each shakes hands with every other person. Prove, by the method of mathematical induction that the total number of handshakes is $\frac{n(n-1)}{2}$.