Comp 2700 (Discrete Structures) Fall 2019. Homework 3.

Submissions: This assignment is due in class on Th Oct 17th 2019. Each student must submit his or her own assignment. Solutions can either be typed in Latex, MSWord or other such word processing software, or printed clearly. You must write your name and UUID clearly on your submitted assignment.

Academic Integrity: You are encouraged to work in groups, but everyone must write out their own solutions. Absolutely no word to word copying is allowed. Please refer to the course policies and schedules about this. If you have worked with other students on the assignment or referred to external sources, please mention all names and sources on your assignment.

Partial solutions: If you are sure that you know how to arrive at a solution, but you get stuck in some place, it is better to write the partial solution. Honest attempts at partial solutions will be awarded.

Name:	
UID:	
Email:	

Problem 1 [20 pts]: For two integers a, b, let $d = (a, b)$ be the gcd of a, b. Show that $(a/d, b/d) = 1$ i.e., the gcd of a/d and b/d is 1.

Problem 2 [5 + 20 + 5 = 30 pts]: In class we have seen a method to solve the modular congruence $ax \equiv b \pmod{n}$ when $(a, n) = 1$. What happens if $(a, n) \neq 1$? Solve the following:

(I) Give an example of a, b, n such that there is no solution to $ax \equiv b \pmod{n}$.

(II) Let $d = (a, n)$. Show that there can be no solution if $d \nmid b$.

(III) Suppose $d|b$. Then show that if an integer x satisfies $ax \equiv b \pmod{n}$ it also satisfies $\frac{a}{d}x \equiv \frac{b}{d} \pmod{\frac{n}{d}}$. Moreover, if x satisfies $\frac{a}{d}x \equiv \frac{b}{d} \pmod{\frac{n}{d}}$ show that it satisfies $ax \equiv b \pmod{n}$. Therefore, we might as well solve the equation $\frac{a}{d}x \equiv \frac{b}{d} \pmod{\frac{n}{d}}$.

(IV) How can you solve the above equation?

Problem 3 [5 + 5 = 10 pts]: Add and multiply $(72531)_8$ and $(4357)_8$ working only in the octal system.

Problem 4 [20 pts]: Let $A = \{1, 2, 4, 7\}$. Give an example of a relation on A that is not symmetric but it is transitive.

Problem 5 [20 pts]: Let $f : \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = x^2 - 2x + 1$. If f injective? Is f surjective? Is it bijective? State your reasons for full points.