Comp 2700 (Discrete Structures) Fall 2019. Homework 4.

Submissions: This assignment is due in class on Th Nov 7th 2019. Each student must submit his or her own assignment. Solutions can either be typed in Latex, MSWord or other such word processing software, or printed clearly. You must write your name and UUID clearly on your submitted assignment.

Academic Integrity: You are encouraged to work in groups, but everyone must write out their own solutions. Absolutely no word to word copying is allowed. Please refer to the course policies and schedules about this. If you have worked with other students on the assignment or referred to external sources, please mention all names and sources on your assignment.

Partial solutions: If you are sure that you know how to arrive at a solution, but you get stuck in some place, it is better to write the partial solution. Honest attempts at partial solutions will be awarded.

Name:	
UID:	
Email:	

Problem 1 [10 + 10 = 20 pts]: Construct the truth table for the following compound propositions:

(a) $(p \leftrightarrow q) \vee (\neg p \leftrightarrow q)$
(b) $((p \to q) \to r) \to s$

Problem 2 [10 + 10 = 20 pts]: Show that the following are tautologies by going through a series of equivalent propositions until you reach T.

(a) $[(p \to q) \wedge (q \to r)] \to (p \to r)$
(b) $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r$

Problem 3 [10 pts]: How many binary strings of length 15 contain the same bit in all the odd numbered positions? The positions are numbered $1, 2, \ldots, 15$. Show how you arrived at your answer, which rules of counting were used etc.

Problem 4 [20 pts]: How many different committees of 6 people can you form from a group of 10 men and 8 women so that it includes 4 women and 2 men? The positions of the committee are not numbered in any way. Show how you arrive at your answer.

Problem 5 [10 + 10 + 10 = 30 pts]: We will try to prove the identity $\sum_{k=0}^{n} k\binom{n}{k} = n2^{n-1}$ using a combinatorial argument.

(I) Think of how you can combinatorially interpret the sum $\sum_{k=0}^{n} k\binom{n}{k}$. Argue that it represents the sum of the sizes of all subsets of an n element set.

(II) Now we count the same number differently. Argue that the above sum is the same as as counting over all elements, how many times an element appears in a subset, i.e., we count for a given element in how many subsets it appears and we sum up the various numbers over all elements. Write an argument as best as you can but also show this for small values of n (say $n \leq 4$) by examples.

(III) Now argue that the number in (II) above is the same for each element (i.e., each element appears in the same number of subsets) and that this is 2^{n-1}. Now finish the argument proving the identity.