Comp 2700 (Discrete Structures) Fall 2019. Homework 5.

Submissions: This assignment is due in class on Tu Dec 3rd 2019. Each student must submit his or her own assignment. Solutions can either be typed in Latex, MSWord or other such word processing software, or printed clearly. You must write your name and UUID clearly on your submitted assignment.

Academic Integrity: You are encouraged to work in groups, but everyone must write out their own solutions. Absolutely no word to word copying is allowed. Please refer to the course policies and schedules about this. If you have worked with other students on the assignment or referred to external sources, please mention all names and sources on your assignment.

Partial solutions: If you are sure that you know how to arrive at a solution, but you get stuck in some place, it is better to write the partial solution. Honest attempts at partial solutions will be awarded.

Name:	
UID:	
Email:	

Problem 1 [20 pts]: An experiment consists of tossing a coin 6 times. Let X be the random variable that is the number of heads in the outcome. Find the mean and variance of X.

Problem 2 [20 pts]: Show from the initial definitions $n + 100 = O(-n^3 + 100n^2)$.

Problem 3 [10 pts]: $n > 5$ balls are thrown into $m > 2$ bins numbered $1, 2, \ldots, m$ uniformly at random i.e., for each ball you toss it into a bin chosen uniformly at random from the m bins. What is the probability that bin 1 receives 2 balls and bin m receives 3 balls?

Problem 4 [20 pts]: Urn A contains 4 red and 7 green balls, and Urn B contains 5 red and 10 green balls. A ball is chosen uniformly at random from urn A and put into urn B. Then a ball is chosen uniformly at random from B. What is the probability this ball is green?

Problem 5 [4 + 8 + 8 = 20 pts]: An experiment consists of rolling ten fair dice each numbered 1 to 6. The outcome is the sequence of numbers obtained. Answer the following questions:

1. What is the size of the sample space of this experiment?

2. Let random variables $X_1, X_2, X_3, X_4, X_5, X_6$ be defined as : X_i is the number of i's obtained among the 10 numbers that show up. What is the expectation, $E(X_1 + X_2 + X_3 + X_4 + X_5 + X_6)$?

3. Let A be the event that exactly five 4's are obtained, and B be the event that exactly five 3's are obtained. What is $P(A \mid B)$? Are A, B independent?

Problem 6 [10 pts]: A hand of 5 cards is dealt out (from a standard pack of 52 cards). What is the probability that there is no ace?